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Angle of approach and angle of recess: What are all these angles?

Considering an often misunderstood, if not completely neglected, set of parameters critical to gear design. If you already understand angles of approach and recess, great. If you do not, read on.

Geometrically speaking, these two angles have a fairly straightforward definition.

The angle of approach is the angle or arc that the point of contact between two mating gears sweeps through prior to becoming coincident with the pitch point; or stated another way, prior to crossing the line between centers. A bit more formal and complete definition is: The arc of approach is the arc along the pitch circle of the gear being analyzed where the tooth profile cuts through the pitch circle of the mating gear when a pair of teeth first come into contact.

The angle of recess is similar, the angle through which the point of contact traverses after it crosses the line of centers (pitch point) to the last point of contact between the two meshing gear teeth. Again, a bit more formally: The arc of recess is the arc of the pitch circle of the gear being analyzed from the pitch point through to where that point of the tooth traverses/crosses the pitch circle of the mating gear.

I now throw a few more definitions into the mix:

- The arc of action is the sum of the arc of approach and arc of recess for any one gear.
- Path of contact (C_P) or contact length is the locus of the point of contact between two mating teeth from the beginning of engagement to the end known as the path of contact or the contact length.
- Path of approach is the portion of the path of contact from the beginning engagement to the pitch point (i.e. length, C_P).
- Path of recess is the portion of the path contact from the pitch point to the end engagement (i.e. length P_D).

Thus, contact ratio can also be derived from the sum of the angle of approach and recess.

So, let's take a look at a graphical representation of the arc of approach, and thus the angle of approach (Figure 1).

The angle of approach is graphically defined as the arc scribed from point *P'* to point *P* in the Figure 1 diagram. Note the direction of rotation of the gear centered at point *A*. The angle of recess is the arc scribed from point *P* to point *P˝*. For reference, the angle φ is the pressure angle for this gear pair.

After a little bit of geometry, you can see that the angle of approach is defined by:

$P'P = (CP)/\cos(\varphi)$

Or length of the line segment point C to point P divided by the cosine of the angle φ.

Similarly, the angle of recess is:

$PP'' = (PD)/\cos(\varphi)$.

Thus, the arc of contact is given by:

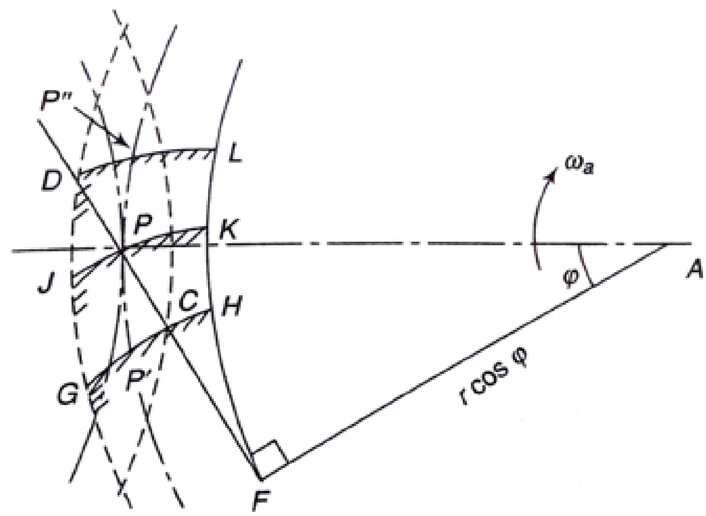

Figure 1

$P'P'' = (CD)/\cos(\varphi)$

OK, enough of the geometry lesson. What is all this good for?

As the instantaneous point of contact in the angle of approach, the tooth-sliding velocity is directed toward the root diameter of the pinion (or the new nomenclature, driver). However, the tooth rolling velocity is toward the outside diameter of the pinion (driver). Through the angle of recess, the rolling velocity continues to be toward the outside diameter of the pinion (driver) and the sliding velocity is also toward the outside diameter of the pinion (driver).

What does this working definition do for us? Well, it helps us understand how to improve, to a certain extent, the lubrication of the gears in mesh without changing the lubricant and/or its viscosity.

How? When the sliding and rolling velocities are opposite to each other, the tooth surface is much more likely to suffer a surface distress failure (pitting). If the sliding and rolling velocities are in the same direction, either up or down the tooth flank, the likelihood of a surface distress failure is reduced. The proof and ability to predict this phenomenon is beyond the scope of this article, but well developed in the standards provided by AGMA and is Dudley's excellent text *Handbook of Practical Gear Design,* to which I owe a great deal of credit for the success I have enjoyed in my career.

As a practical matter, it is recommended that a gear designer consider developing a design that has a longer arc of recess than it does approach. This becomes more critical to good performance of the geartrain and longer predicted service life as the difference in diameter of one to gear to the other (driver to driven for either an under drive or an overdrive design). This recommended practice makes common sense as well since the smaller diameter gear will be rotating multiple times as fast as the larger gear and each tooth sees significantly more load cycles than its mate.

There are many techniques and tools that can assist a designer to develop suitable distribution between the arc of approach and the arc of recess as a function of the life requirements, etc. For most tools, the analytic methods are robust and accurately predict both the geometry and sliding velocity distribution. I recommend these values be studied and manipulated, but I would also recommend looking at the sliding velocity graphically. I will not attempt to provide either exact or rule-of-thumb values as a function of the gear-design parameters. You can develop your own "best practices" by reverse engineering both successful geartrains and geartrains that did not provide the anticipated service life. From this analysis, one can develop their own ratio of approach to recess action based on their company standard design practice, product environment, lubricant used, etc. As you all know by now, I am all about lubrication. Yes, good design practice, good materials, good heat treatment (if appropriate), and post-heat treatment finish machining are critical. However, as I have stated many times, a great geartrain will fail prematurely without adequate lubrication, and a "poor" gear design can be positively augmented through a well-developed lubricant and lubrication system.

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ABOUT THE AUTHOR

Dr. William Mark McVea, P.E., is President and Principal Engineer of KBE+, Inc. which develops complete powertrains for automotive and off-highway vehicles. He is the Principal Engineer with Kinatech, a joint venture with Gear Motions / Nixon Gear. He has published extensively and holds or is listed as co-inventor on numerous patents related to mechanical power transmissions. Mark, a licensed Professional Engineer, has a B.S. in Mechanical Engineering from the Rochester Institute of Technology, a Ph.D. in Design Engineering from Purdue University.